Local Spatial Rules which Determine Cities Generation

Ph.D. Thesis Synopsis

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City is an emergent and evolutionary complex system. Finding objective rules that drive city generation process and defining their roles will help more objective explanation of urban patterns and more predictable decision makings in cities. This research expresses existence of objective rules in city generation process and intends to define some of these rules which are spatial in nature.

This research considers a city as a complex system consisting of open and closed spaces formed by arrangement of buildings. These buildings are gradually located in space by people decided to gain the maximum benefit of the environment based on the local predictability provided by the physical reality without following any predefined plans defined by a third party. It defines an organic unplanned pedestrian city, i.e. a small city. The hypothesis is "*the physical reality provides a concrete set of local spatial rules to optimally locate buildings accumulate to generate a city*."

A computational model is developed studying the hypothesis. The model consists of a set of rules, a mechanism for combining the rules and simulating their interactions, and algorithms for measuring some global urban patterns. Using this model, the following three questions are answered:

Q1. What is the minimum information required for a man to locate a suitable place for a building to live?

A1. The minimum rule set defined in this research contains 7 simple spatial rules including (1) Distance-to-Center-of-Gravity, (2) Distance-to-Road, (3) Free-Space, (4) Adjoining-Free-Space, (5) Access-Space, (6) Adjoining-Access-Space, and (7) Sun-Position. Application of these rules just requires the basic abilities to recognize proximity and neighborhood.

Q2. What is the decision making process for combining these rules i.e. what is the simulation process?

A2. Each simulation of the model starts by adding one or more buildings as seeds, and perhaps some roads in the environment. These entities can be imported into the model from a map or drawn and edited by the operator.

At each step of the simulation, one building and its required free space is added to the environment. The locations of the building and its free space are defined by applying the rules to the existing spatial arrangement of the entities (roads, buildings, and free spaces) in the environment. Accumulation of the buildings and their free spaces generates the expected city environment and patterns.

At first, location of the building (the closed space) is defined using Distance-to-Center-of-Gravity and Distance-to-Road rules, which interact with the environment at a global scale. One of the cells with the maximum normalized value is selected randomly.

Location of the required free-space for the selected building is defined by applying the rest of the rules follow this order: 1) Free-Space, 2) Adjoining-Free-Space, 3) Access-Space, 4) Adjoining-Access-Space, and 5) Sun-Position. These rules interact with the environment at a local scale. They are applied in the order enumerated. The process of locating the free space stops when one possible cell is remained. If more than one possible cell is remained, after the rules are applied, one of the cells is selected randomly. The Sun-Position rule used, however, discards the random selection of free spaces and ensures that the possible locations for a free-space are always reduced into one cell.

The Sun-Position rule interacts at a local scale, although it has global/similar effect on all the locations. Avoiding dominance of the global effect of the Sun-Position rule, it is applied as the last rule, if more than one possible cell is remained.

Q3. Dose this knowledge suffice to generate a city by accumulation of these optimally located buildings?

A3. Three universal global patterns are studied using the computational model developed in this research including (1) The small-world network pattern, (2) The rank-order distribution of axial lines length following the power law pattern, and (3) fractal pattern. It is shown that these properties of the patterns generated using the model are similar to their real counterparts observed in cities and the generated patterns are statistically stable.

The small-world network pattern is characterized by *p* that defines the probability of a node to be connected to the further nodes (Not the neighbor nodes). In real cities the value of *log(p)* ranges from 0.03 to 0.09 which covers the p value of the generated pattern. The value of *log(p)* for the generated pattern ranges from 0.05 to 0.09.

The rank-order distribution of axial lines length pattern is characterized by the exponents of the power law distributions fitted to its truncated and un-truncated distributions their inverses denoted as alpha and z-alpha. In real cities, the alpha decreases towards 2 and the z-alpha increases towards 1 when local structure of space dominates. The alpha exponent of the generated pattern is around 2.684 (with stddev = 0.316) and its z-alpha is round 1.45 (with stddev = 0.11) that represent the significant role of the local rules, causing the emergence of more short axial lines.

The fractal pattern is characterized by fractal dimension denoted as *D*. In real cities the value of *D* is around 1.7. The D for the generated pattern is around 1.65.

It is shown that the generated pattern and the mentioned characteristic values are statistically stable using different initial values. It represents the required universality of the generated patterns like the three patterns studied.

The discussions admit that the global rules lead cities toward generation of longer and less fractured passages. It represents the existence of a rather steady growth of free spaces in the simulated environments. The local rules, however, break this steady growth in favour of faster changes at a local scale. The model still allows for slow and steady growth at a global scale, however. The effects cause the emergence of a hierarchical environment. Such an environment supports the generation of life-like structures like cities.

The patterns generated in the model as the result of local interactions of the seven simple spatial global (2 rules) and local rules (5 rules). The effects of the Access-Space and the Adjoining-Access-Space local rules are more significant in altering the global spatial structure, causing the emergence of alley-like passages around the roads (Fig. 3).

The other results of the research are the algorithms implemented for measuring the properties of the three patterns studied. These algorithms are (1) Automatic generation of connection graph and computing the clustering and mean-shortest-path value of the graph, (2) Automatic extraction of axial map and regression of the length of axial line to the power law distribution, and (3) Automatic measurement of the fractal dimension using box-counting method. All these algorithms are implemented for the raster representation of space generated by the simulation model.

The further work suggested are regeneration of other global urban patterns which may require extending the rule-set defined here and increasing the efficiency of the methods developed for measurement of the 3 urban patterns studied in this research and investigating the rasterization effect on these methods.

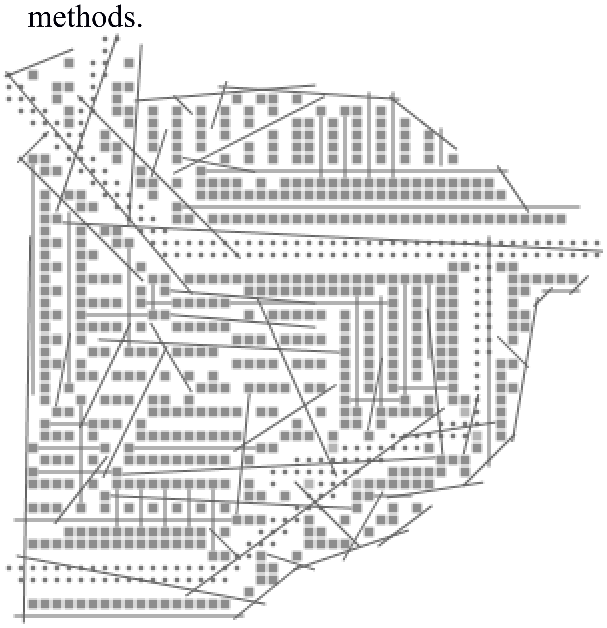

Fig. 3. A sample simulation using all the rules. Boxes represent buildings/closed-spaces, dots represent passages/free-spaces, and the lines represent the axial lines.

Appendix A – The Global Urban Patterns

This section summarizes the specifications of the patterns studied in this research. The following sections are excerpts from the following papers:

1. Rezayan, H., Delavar, M. R., Frank, A. U., & Mansouri, A. (2008). Spatial Rules Generate Urban Patterns: Emergence of the Small-World Network. *Lecture Notes In Geoinformation and Cartography (LNG&C): Headway in Spatial Data Handling, 13th International Symposium on Spatial data Handling*, 533-556.
2. Rezayan, H., Delavar, M. R., Frank, A. U., & Mansouri, A. (2010a). Spatial rules that generate urban patterns: Emergence of the power law in the distribution of axial line length. *International Journal of Applied Earth Observation And GeoInformation (accpeted to be published)*.
3. Rezayan, H., Delavar, M. R., Frank, A. U., & Mansouri, A. (2010b). Spatial rules that generate urban patterns: Emergence of the Fractal Urban Pattern. *Under Review*.

The Small-World Network

Small-world network is a class of random networks that its nodes are connected by both long and short links (Salingaros, 2004). Then each node in the small-world network can reach most of the other nodes by a small number of steps, although most of the nodes are not neighbors (Fig. A.1).

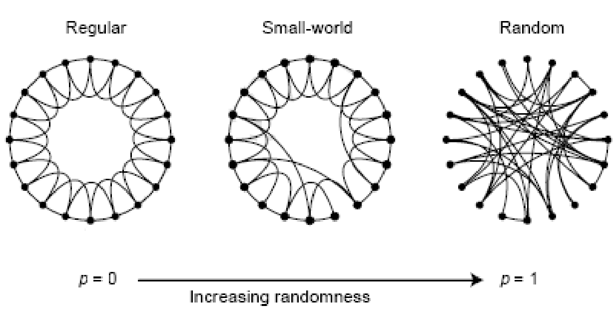

Fig. A.1. The small-world network stands between a regular and a random network. It is highly clustered like a regular graph, yet with small characteristic path length, like a random graph. Here *p* represents the probability of connecting a node/vertex to far nodes/vertices, rather than immediate nearest neighbor nodes. It is defined by node's degree. In this figure, nodes have 4 degrees (Watts and Strogatz, 1998).

Watts and Strogatz (1998) compared the mean-shortest path length and the clustering coefficient of regular, small-world, and random networks. They define the mean-shortest path length as "… the number of edges in the shortest path between two vertices, averaged over all pairs of vertices. (Watts and Strogatz, 1998)" The clustering coefficient is also defined as the average of edges exists between neighbours of a vertex to maximum number of edges between them over all vertices. The maximum number of edges that can exist between the n vertices is $n*(n-1)/2$ for an undirected network.

Watts and Strogatz (1998) showed that the mean-shortest path length of a small-world network and random networks are similarly small, but the clustering coefficient of a small-world network is larger than what is expected for random networks (Fig. A.2). It means that a small-world network has few high degree nodes, known as hubs, and the rest of the nodes are peripheral, low degree nodes. It brings stability against changes may happen in the peripheral nodes. It makes the small-world network pattern reliable enough to support life of networks like World Wide Web or a city.

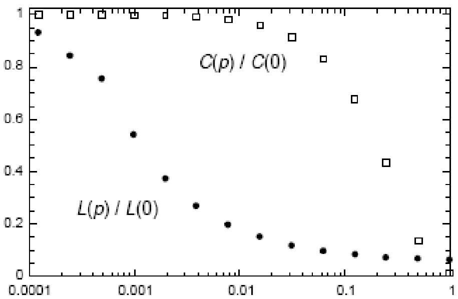

Fig. A.2. The mean-shortest path length *L(p)* and the clustering coefficient *C(p)* for the family of randomly rewired graphs with 1000 nodes which have 10 degrees. The x-axis represents the probability of networks (*p*) in logarithmic scale. The values are normalized using *L(0)*, *C(0)* for a regular lattice. (Watts and Strogatz, 1998)

Degree-distribution of the small-world network fits the power-law distribution. It means that the small-world network is scale-free. The small-world network also encourages movement for it inherits the predictability of regular networks and accessibility of random networks.

For a network of spaces in a city, the mean-shortest path length represents how far you should go to be able to reach urban facilities like stations and shops. The clustering coefficient reflects how stable and reliable these accesses are, considering the continuous changes may happen due to human activities, like accidents or constructions, or environmental conditions (e.g. bad weathers) that can hinder or block normal flows in a city. These are the basic characteristics of an urban structure, which is alive (Salingaros, 2003).

Distribution of axial line length according to power law

Hillier and Hanson (1984) define an axial line as the longest straight line that can be drawn from an arbitrary point in space. Axial lines represent convex spaces. They connect together to form an axial map that is a geometrical representation of space.

In studying the pattern of free spaces and intermittency in cities, Carvalho and Penn (2004) evaluate the rank-order distribution of axial line length (Eq. A.1) in 36 cities from 14 countries according to the power law (Fig. A.3). Following the power low means that the distribution is scale-free and has hierarchical structure emerged through a bottom-up process of gradual generation.

$\ln(size) \zeta = b\ln(\ * \ ) rank$	Eq. A.1

where *size* is the length of an axial line divided by the average lengths of axial lines (length/avg(lengths)),

rank is the ranking of the axial line, within a list of the axial lines sorted in descending order of length, which is then divided by the maximum rank of all axial lines (rank/max(ranks)),

ζ (zeta) is the slope of the trend line, and

b is the y interception of the trend line.

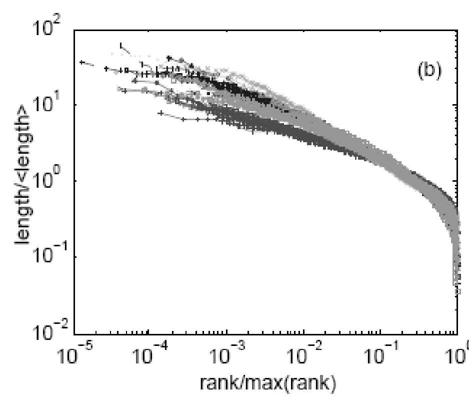

Fig. A.3. **[a]** The rank-order distributions of axial line length in 36 cities from 14 countries (Carvalho and Penn 2004, p. 10). The axes have a logarithmic scale. The dashed line shows the truncation line at ln(length/<length>) = 0.2 or length/<length> = 1.221 where <length> represents average length.

Carvalho and Penn (2004) fit the Eq. A.1 to the upper tail of the distribution truncated at ln(length/avg(length)) = 0.2 or length/avg(length) = 1.221 (Fig. A.3) where "… data are visually the most linear (Carvalho and Penn 2004, p. 4)". They state that "the length of urban free space structures represented by axial lines, display universal features, largely independent of city size, and is self-similar across morphologically relevant ranges of scales with [alpha] exponents 2 and 3 (Fig. A.4)." (Carvalho and Penn 2004, p. 4). The alpha exponent is defined as the inverse of the zeta exponent (Eq. A.2).

$\alpha = 1/\zeta$	Eq. 2

where ζ (zeta) is the slope of the trend line shown in Eq. A.1.

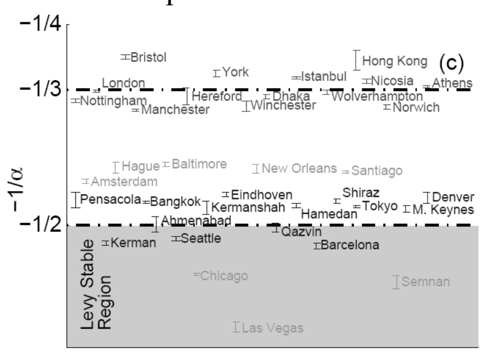

Fig. A.4. Distribution of the inverse of the alpha exponent (Eq. 2) (or zeta exponent in Eq. A.1) derived from the graph shown in Fig. A.3 (Carvalho and Penn 2004, p. 10). The stable region of the alpha exponent is shown in gray where the alpha exponent is lower than 2.

The alpha exponent decreases (Fig. A.3) as the distribution of the axial line length leans more toward longer axial lines (Fig. A.4). Carvalho and Penn (2004) discuss this effect as an increase in the dominance of the global spatial structure (Fig. A.4). The increase of the alpha exponent represents an increase in the dominance of the local spatial structure. This effect can be represented using the median length of axial lines. The existence of more short axial lines moves the median length of axial lines toward their minimum length and vice versa.

Fractal Pattern of Cities

"Fractals, a term coined by its originator Benoit Mandelbrot (1983), are objects of any kind whose spatial form is nowhere smooth, hence termed irregular, and whose irregularity repeats itself geometrically across many scales. In short, the irregularity of the form is similar from scale to scale, and the object is said to possess the property of self similarity or scale invariance. (Batty and Longely 1994, p. 3)"

A fractal pattern is specified by its dimension also denoted as *D*. It measures the similarity of different levels of space, or self-similarity, and the space filling property of a pattern. Higher fractal dimension represents more fractured geometry that fills the space more and vice versa.

Different definitions are presented for fractal dimension based on the methods used for measurement of fractal dimension. Although these definitions usually result in very similar fractal dimensions for a pattern, each definition and its measurement method are computationally suitable for a specific kind of structure. For example the similarity and the geometrical methods of measuring fractal dimension are suitable for regular patterns while the box-counting and the correlation methods are more appropriate for irregular patterns like urban structures. Between the box-counting and the correlation method, the box-counting is simpler for measuring the fractal dimension of urban patterns (Batty and Longely 1994).

The box-counting method is used in this paper. It is based on counting the number of boxes cover a pattern at different scales (Eq. A.3).

$D = Log(N)/\log(1/s)$	Eq. A.3

Where *D* is the fractal dimension,

N is the number of boxes required to cover a pattern, and

s is the scale in which the measurement is carried out.

The similarity of fractal dimension at different scales shows how self-similar is the structure of a pattern and the processes that generated it. Ideally, fractal dimension of a fractal pattern is similar at different scales and equals the ratio of *Log(N)* to *Log(1/s)*. It represents complete self-similarity in the pattern.

In reality, fractal dimension changes from scale to scale and the ratio of *Log(N)* to *Log(1/s)* is not similar at different scales. In this case the slope of the line fitted to the distribution of *Log(N)* to *Log(1/s)* is considered as fractal dimension. Sum of the squared residuals (*SSRES*) of the fitted line represents the amount of the changes or the stability of the fractal dimension.

The similarity of fractal patterns in cities and their fractal dimensions are investigated by Anas et al. (1998). They evaluated a number of large cities and concluded that their fractal dimension is about 1.7.

Although a specific range for fractal dimension of urban structures is not accepted, the pervasiveness of fractal patterns in cities is admitted (Salingaros 2003, Batty and Longely 1994, Anas et al. 1998). The fractal properties might also change through the time during the generation and evolution of cities (Batty and Longely 1994, Benguigui et al. 1999). In a period a city might grow following fractal pattern but not in another period. The periods which a city does not follow fractal pattern correlates with the periods that city plans are executed in favor of creating long links, like main roads and highways, and destroying short links especially pedestrians (Salingaros 2003).

"Only older, pre-modernist cities are [completely] fractal, because they work on all scales (Salingaros 2003)." These are organic, unplanned pedestrian cities, mostly small cities. They are also called pedestrian cities (Salingaros 2004) or car-free cities (Crawford 2002) as their structures foster pedestrian movement. This structure can also be founded in surviving historical regions of cities, commercial areas, some suburbs, and urban-villages (Fleming 2000, Spears 1997), where pedestrian passage is the main medium of movement.