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Abstract

Simplicial complexes are a standard mathematical device underlying many data structures in computational geometry (Paoluzzi and others 1993). They serve to represent geometric objects and configurations by collections of elementary building blocks for each dimension (nodes, edges, triangles, tetrahedrons etc.), observing special intersection constraints (Giblin 1977).

Application-specific data models based on simplicial complexes have been developed for various domains. Among them is the Simplicial Data Model (Egenhofer and others 1990; Frank and Kuhn 1986), designed primarily for the management of vector data in geographic information systems (GIS). This model allows for a consistent representation of topological relationships. Inconsistencies can, however, arise from repeated intersections of line segments, due to the finite resolution of computational number systems.

The present work extends the Simplicial Data Model to discrete coordinate spaces. The result is a robust specification and implementation of plane simplicial complexes for finite representations. We start from Greene and Yao's (Greene and Yao 1986) method for line segment intersections in the discrete domain. This method guarantees that intersection points remain contained within small envelopes around the original line segments. However, it cannot resolve certain ambiguities and the resulting representation of topological relationships depends on the sequence of object insertions.

Our approach retains the idea of envelopes, avoiding the artifact of wandering lines, but replaces Greene and Yao's redrawing of line segments by a hierarchical representation of line segments and their subdivisions. In addition to the stronger consistency requirements, this also satisfies the need for retracing the history of line segment subdivisions as well as for keeping track of the collinearity of line segments.

The presentation will highlight the key ideas of our method and demonstrate an algebraic specification and prototype implementation in the functional programming language GOFER.

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